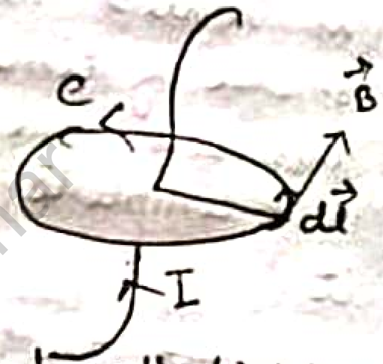
Displacement Current

Ampere's circuital law in most general form is given by

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 \iint_S \vec{J}\cdot d\vec{s} \qquad -① $$



where $\vec{B}$ is the magnetic induction due to a current

$$I = \iint_S \vec{J}\cdot d\vec{S} \qquad -② $$

in a conductor and C is closed path linking current I. $\vec{J}$ is the current density and s is the cross-sectional area of the conductor.

By Stokes' law

$$\oint \vec{B}\, d\vec{l} = \iint_S (\nabla\times\vec{B})\cdot d\vec{S}$$

$$\therefore \iint_S (\nabla\times\vec{B})\cdot d\vec{S} = \mu_0 \iint \vec{J}\cdot d\vec{S} \qquad -③ $$

$$\text{or} \quad \nabla\times\vec{B} = \mu_0\vec{J} \qquad -④ $$

Taking divergence on both side

$$\nabla\cdot(\nabla\times\vec{B}) = \mu_0(\nabla\cdot\vec{J})$$

Since $\nabla\cdot(\vec{\nabla}\times\vec{B}) = 0$ & $\mu_0 \neq 0$

$$\therefore \nabla\cdot\vec{J} = 0 \qquad -⑤ $$

Now the equation of continuity is

$$\nabla\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0 \qquad -⑥ $$

where ρ is the charge density

from ⑤ $\quad \frac{\partial\rho}{\partial t} = 0 \quad [\because \nabla\cdot\vec{J} = 0]$

where ρ = constant of time.

This means that Ampere's circuital law is valid only in case where charge density is static i.e. steady state condition of charge flow so that $\rho = 0$. Thus Ampere's circuital law, as stated above, in case where $\frac{\partial \rho}{\partial t} \neq 0$, i.e. in case of time dependent field does not hold good. This led Maxwell to assume that equation ① is not complete and some thing is to be added to it.

Let the quantity to be added to the right hand side of equation ① $\vec{J}_D$, then

$$(\nabla \times \vec{B}) = \mu_0 (\vec{J} + \vec{J}_D) \quad \text{—— ②}$$

Taking the divergence on both sides

$$\nabla \cdot (\nabla \times \vec{B}) = \mu_0 \nabla \cdot (\vec{J} + \vec{J}_D)$$

$$0 = \mu_0 (\nabla \cdot \vec{J} + \nabla \cdot \vec{J}_D)$$

or $$\nabla \cdot \vec{J}_D = - \nabla \cdot \vec{J}$$

$$= \frac{\partial \rho}{\partial t} \quad \left[\because \vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0\right]$$

$$= \frac{\partial}{\partial t} (\nabla \cdot \vec{B}) \quad [\because \rho = \nabla \cdot \vec{D}]$$

$\vec{B}$ is the electric displacement vector

$$\therefore \nabla \cdot \vec{J}_D = \frac{\partial}{\partial t} (\nabla \cdot \vec{D}) = 0$$

$$\nabla \cdot \left[\vec{J}_D - \frac{\partial \vec{D}}{\partial t}\right] = 0$$

$$\therefore \vec{J}_D = \frac{\partial \vec{D}}{\partial t}$$

Now the modified form of Ampere's circuital law takes the form

$$\nabla \times \vec{B} = \mu_0\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$$

$\vec{J}_D = \frac{\partial \vec{D}}{\partial t}$ is known as displacement current density which is different from charge transported current density $(\vec{J})$

Important features of displacement current:

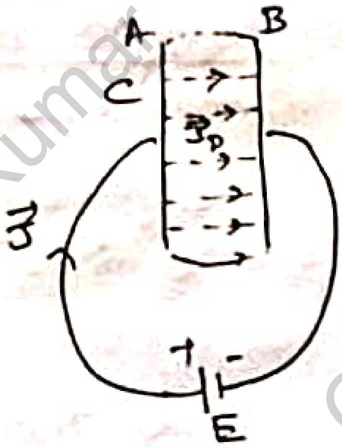
(i) Displacement current is a current only in the sense that it produces a magnetic field. It has none of the other properties of current as it is not related to motion of charges.

(ii) The magnitude of $\vec{J}_D$ is equal to the time rate of change of displacement vector $\vec{D}$. $\vec{J}_D$ may have a certain value in vacuum, where $J = 0$ due to absence of charge.

(iii) Displacement current makes the total current continuous across the discontinuity in conduction current.

(iv) In a conductor, the displacement current is negligible as compared to the conduction current.

The displacement current $\vec{J}_D$ and conduction current $\vec{J}$ are shown in a simple circuit containing a condenser C and an source of e.m.f E.



$$\vec{J}_{total} = \vec{J}_D + \vec{J}$$